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Design Considerations of an FSAE Steering System

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Washington University in St. Louis

JAMES MCKELVEY SCHOOL OF ENGINEERING

Fall 2018 MEMS 400 Independent Study Project

Design Considerations of an FSAE Steering System

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General Background

WashU Racing

Consisting of 80 members as of the 2019 season, WashU Racing is an engineering group that designs, manufactures, and assembles an open-wheel racecar over a year. With the end goal of competing in FSAE Michigan, the team strives to improve the car's quality every year. The team consists of around 80 students from all the engineering disciplines, as well as physics, art, and business students that contribute to both design and project management. These students work long weekdays and even longer weekends to assemble a vehicle that requires as much thought and effort as a real world engineering project.

The team is structured with an executive group and a system leads group. The executive group works constantly to manage the project and promote communication between the team and sponsors. This group consists of a President, Vice President, Chief Engineer, Treasurer, and a Recruiter. Under this group, there are the system leads, each of which are in charge of a specific system of the car. These systems are Frame/Chassis, Suspension, Body and Aerodynamics, Powertrain, Drivetrain, Electronics, Data Acquisition, Ergonomics, and Manufacturing. The system leads use their expertise to guide their system's members in designs and fabrication all while designing their own parts and subsystems. This team structure is the result of iterating through teams ever since the team's revival in 2011, which, this year, has now resulted in the team's decision to change the project's public platform from BFR (Bear Formula Racing) to WUFR (Washington University Formula Racing), signaling a positive change with a focus in justified design and constant thought in the build process.

FSAE Competition

FSAE Competitions consist of multiple events which contribute to a total of 1000 points. The events are divided into two groups of static and dynamic, with 325 points allotted to static events, and 675 points to dynamic events. For the dynamic events there is a limit of two drivers [6].

Static Events

Presentation

In the business presentation, our business team will present to a panel to convince theoretical buyers to invest in the team and the car. This event is worth 75 points and demonstrates our confidence in the car and our ability to advertise our engineering skills to an audience of interest. This event has no effect on the design of our steering system.

Cost Analysis

The cost analysis is a submitted report of the build cost for each car. It requires a bill of materials for the entire car as well. While the cost of the steering system is important, the analysis is outside the scope of this project.

Design

The design competition is arguably the most important portion of the competition, where teams get to pitch their car and justify their engineering choices to industry engineers. In this event, our steering designs will need to be rigorously defended and justified to the design judges. This event is worth 150 points, which is the largest amount of points for any single event except for the endurance event. More importantly, this event showcases the knowledge and confidence we have in our designs.

Dynamic Events

Acceleration

As the name suggests, the Acceleration event is a test of each car's straight-line acceleration on a 75 m course. Each car begins 0.3 meters behind the starting line. As soon as the car crosses the starting line the timer begins, and as soon as it crosses the finish line the timer ends. There is a limit of four trials, two per driver.

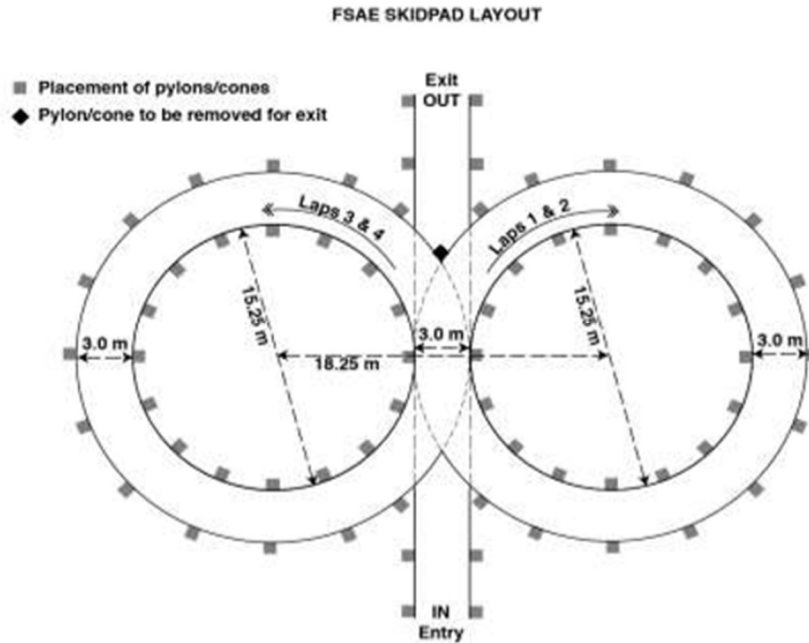


Figure 1: FSAE skid pad layout

Skid Pad

The skid pad event is a test for the vehicle's lateral acceleration in a flat corner with a constant radius. Following the course shown in Figure 1, the cars are to follow a path around two circles, each with an internal diameter of 15.25 meters. There is a limit of four trials, two per driver, same as in the acceleration event.

Autocross

The Autocross event is a small scale version of the Endurance event. It consists of a single lap around a course length of 0.80 km. It has the purpose of testing the vehicle's handling in order to place the car's starting point in the Endurance event. The course design has several parameters for each track feature:

- Straights: No longer than 60 m with hairpins at both ends
- Straights: No longer than 45 m with wide turns on the ends
- Constant Turns: 23 m to 45 m diameter
- Hairpin Turns: 9 m minimum outside diameter (of the turn)
- Slaloms: Cones in a straight line with 7.62 m to 12.19 m spacing
- Miscellaneous: Chicanes, multiple turns, decreasing radius turns, etc.
- Minimum track width: 3.5 m
- Length of each run should be approximately 0.80 km

Efficiency

The Efficiency event happens in coordination with Endurance. The car is completely refueled before endurance and the fuel loss after the Endurance event is measured to get the vehicle's fuel economy.

Endurance

The Endurance event is the culmination of the competition, the final test for each vehicle's maneuverability, drivability, and speed. Drivers will drive along a 22 km course consisting of multiple laps. The track features have a slightly different ruleset from the Autocross event:

- Straights: No longer than 77 m with hairpins at both ends

- b. Straights: No longer than 61 m with wide turns on the ends
- c. Constant Turns: 30 m to 54 m diameter
- d. Hairpin Turns: 9 m minimum outside diameter (of the turn)
- e. Slaloms: Cones in a straight line with 9 m to 15 m spacing
- f. Miscellaneous: Chicanes, multiple turns, decreasing radius turns, etc.
- g. Minimum track width: 4.5 m
- h. Designated passing zones at several locations

FSAE Steering Rules

Outside of the actual competitive events, the car must first pass a technical inspection. This is the most rigorous and detailed test for the car, and sets some of the most critical design limitations. The rules that the steering system needs to satisfy to pass technical inspection are listed below:

T.1.6 Steering

T.1.6.1 The steering wheel must be mechanically connected to the front wheels.

T.1.6.2 Electrically actuated steering of the front wheels is prohibited.

T.1.6.3 Steering systems using cables or belts for actuation are not permitted

T.1.6.4 The steering system must have positive steering stops that prevent the steering linkages from locking up (the inversion of a four bar linkage at one of the pivots). The stops may be placed on the uprights or on the rack and must prevent the wheels and tires from contacting suspension, body, or frame members during the track events.

T.1.6.5 Allowable steering system free play is limited to seven degrees (7°) total measured at the steering wheel.

T.1.6.6 The steering wheel must be attached to the column with a quick disconnect. The driver must be able to operate the quick disconnect while in the normal driving position with gloves on.

T.1.6.7 The steering wheel must have a continuous perimeter that is near circular or near oval. The outer perimeter profile may have some straight sections, but no concave sections. "H", "Figure 8", or cutout wheels are not allowed.

T.1.6.8 In any angular position, the top of the steering wheel must be no higher than the top-most surface of the Front Hoop. See T.2.13.4*

T.1.6.9 The steering rack must be mechanically attached to the frame

T.1.6.10 Joints between all components attaching the steering wheel to the steering rack must be mechanical and be visible at Technical Inspection. Bonded joints without a mechanical backup are not permitted.

T.1.6.11 Fasteners in the steering system are Critical Fasteners, see T.10.2* and T.10.3*

T.1.6.12 Spherical rod ends and spherical bearings in the steering must meet T.1.5.5* above

T.1.6.13 Rear wheel steering may be used.

- Rear wheel steering must incorporate mechanical stops to limit the range of angular movement of the rear wheels to a maximum of six degrees (6°).
- The team must provide the ability for the steering angle range to be verified at Technical Inspection with a driver in the vehicle.
- Rear wheel steering may be electrically actuated.

**Supplementary rules can be seen in Appendix A*

Function Tree and General Design

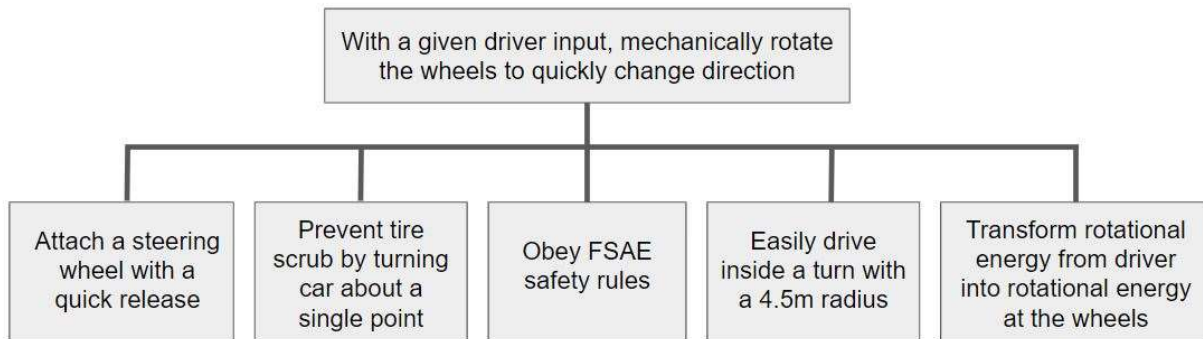


Figure 2: Steering System Function Tree

The most basic job of a steering system is to simply rotate wheels given an input. However, an FSAE steering system needs to do much more, which can be seen in Figure 2 above. For this design, the rotational force given by the driver follows the flowchart shown below in Figure 3.

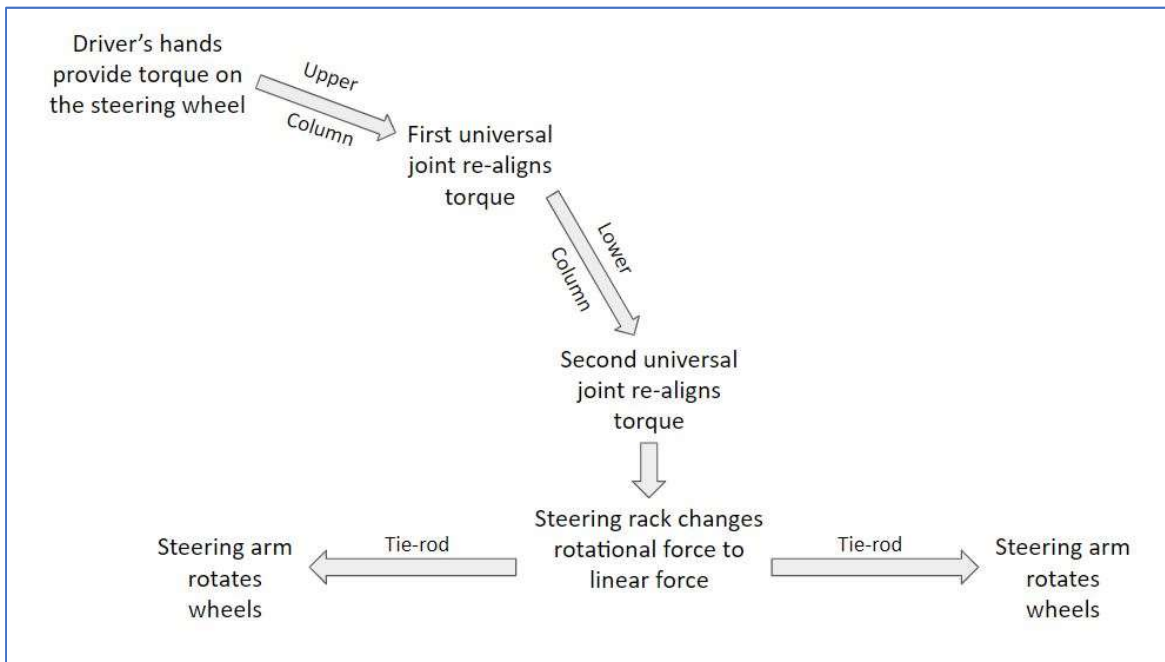


Figure 3: Steering Force Flowchart

Goals and Limitations

The goals and limitations of this steering system can be classified into three categories: design-, driver-, and packaging-defined.

Design Defined

Many goals are based on optimizing and choosing sound designs. For example, the FSAE rules act as strict design limitations. Design-based goals are the starting point for this design process, and can be edited later in accordance to the constraints from the driver and packaging goals and limitations. The most prominent design defined goals are as follows:

- Easily make a turn with an outside radius of 4.5 meters
- Eliminate tire slipping around corners at low speeds
- Place all components in positions that place the least amount of deflection and stress on the part

Driver Defined

Driver-defined goals are those which are a direct result of driver feedback. For the 2019 year, WashU Racing has placed an overall design focus on constructing a drivable car. This means the driver needs to feel confident in the car and its handling at all times, and this directly relates to the design of the steering system. The driver is given a large amount of their road information from the forces in the steering wheel, and so the force path between the wheels and the steering wheel must be sufficient enough that any outside interference is limited. A few goals and limitations defined by the driver are shown below:

- Steering wheel must have 110° to each side to guarantee maximum wheel turn without the driver removing their hands from the wheel
- The force required to steer must equal to or less than the BFR-18's steering force
- The steering column must utilize two universal joints to provide consistent feedback from the road
- The steering rack must be mounted at the ends to reduce compliance in the system.

Packaging Defined

The final limitation factor for this design process is the packaging. WUFR-19 has introduced a new challenge that has never been seen in WashU Racing: smaller wheels. This smaller wheel size drastically impacts everything that will be placed inside the wheel, especially the steering arm adapter. These limitations will be the final check for all designs and parts.

Steering Column

Universal Joints

Properties of U-Joints

Unless the steering wheel axis is directly aligned with the pinion axis of the steering rack, some sort of mechanical joint is needed to angle the rotational force along a new axis. The conventional method of transferring force through such an angle is by the use of either a single universal joint (or u-joint for short), double universal joint, or two universal joints. The difference between the latter two methods are where they are constrained, resulting in separate output axis locations. Universal joints must be mounted properly to function: the rotational shafts must be constrained radially, but free to rotate, therefore calling for a bearing. The three configurations are shown below in Figures 3,4, and 5.



Figure 4: Single u-joint



Figure 5: Double u-joint



Figure 6: Two u-joints

For these configurations, the system must be constrained radially about the input and output shafts. However, due to small imperfections in attaching the shafts, it is best to have these constraints located immediately before the input shaft-universal joint connection and immediately after the output shaft-universal joint connection. Figure 7 below shows these locations in the two-universal joint configuration.



Figure 7: U-joint constraint locations

This design will use the two universal joints configuration for two reasons. First, due to driver preferences, the more vertical steering wheel angle requires a greater angle between the upper steering axis and the steering pinion axis, which can only be satisfied with two universal joints. Secondly, a double universal joint requires a rotational constraint much higher up the steering axis. Using the two universal joints allows for the attachment point between the lower u-joint, allows the steering rack to act as the radial constraint, and provides optimal feedback from the road through the steering column.

The actual steering wheel placement will require two of these universal joints, which benefits the design by providing a greater range for the steering wheel angle, but at the same time adds weight and complexity to the system. However, because the steering rack will be closer to the driver than previous years, it requires a greater angle range to avoid a steering wheel that is mounted too flat.

The most important benefit of using two u-joints is that the second u-joint counteracts any negative effects which the first one might have with driver feel. Universal joints also have one large drawback: at a given input

shaft angular velocity, the output shaft angular velocity fluctuates. This fluctuation can be described by the following ratio...

$$\frac{\omega_o}{\omega_i} = \frac{\cos \gamma}{1 - \sin^2 \gamma \cos^2 \theta}$$

...where ω_o is the angular velocity of the output shaft (rad/s), ω_i is the angular velocity of the input shaft (rad/s), γ is the angular misalignment of the input and output shaft in radians, and θ is the angular position of the input shaft relative to the input section of the u-joint.

It can be seen that as the angle between the shafts γ increases, the ratio of output velocity to input velocity fluctuates, and as γ approaches zero the fluctuations become negligible as the entire equation goes to 1. This dependency on the axis misalignment can be seen below in Figure 8 (See Appendix B for source code).

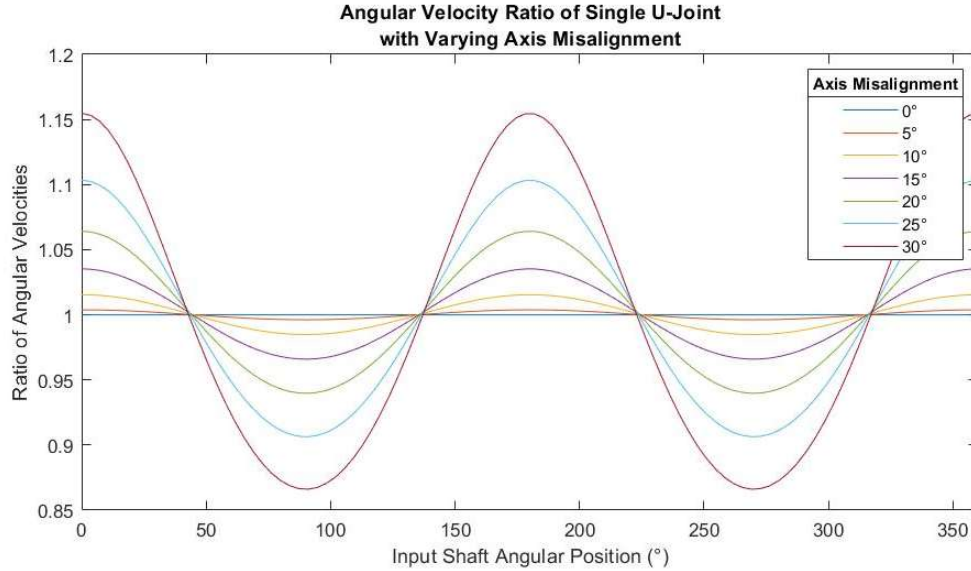


Figure 8: U-joint output velocity variation graph

This design's chosen u-joint, purchased from the same manufacturer as the steering rack, has an absolute operation limit of 32°, and a recommended limit of 20° for which the fluctuations will be negligible. This information might lead one to believe that this design can operate at a maximum angle of 40°. However, the addition of a second u-joint allows for the first to be canceled out.

By adding a second u-joint and altering the new θ term, the fluctuations due to each universal joint are able to cancel each other out. This allows a constant output angular velocity from a constant input joint, transforming a u-joint into a constant velocity joint. This constant velocity is important for the car's driver to feel any forces from the road with greater accuracy. Figure 9 shows the effect of altering the second u-joint's θ term with two universal joints at an axis misalignment of 20°.

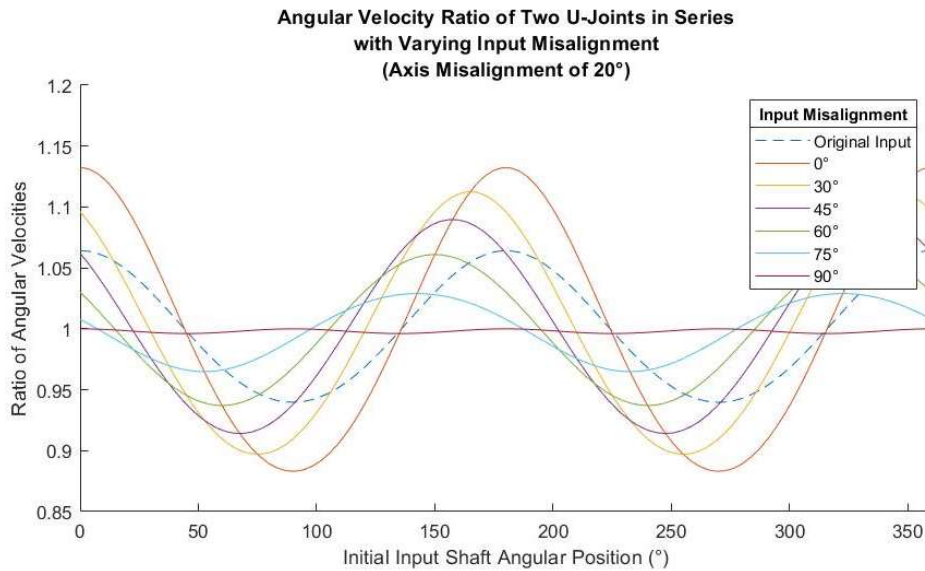


Figure 9: Second u-joint dampening graph

Upper Steering Column

The upper steering column serves the purpose of constraining the upper u-joint and connecting the u-joint splines with the steering wheel quick release. Figure 10 below shows an exploded view of the assembly, with Figure 11 showing the cross section view of the assembly. Table 1 shows the bill of materials with part descriptions.

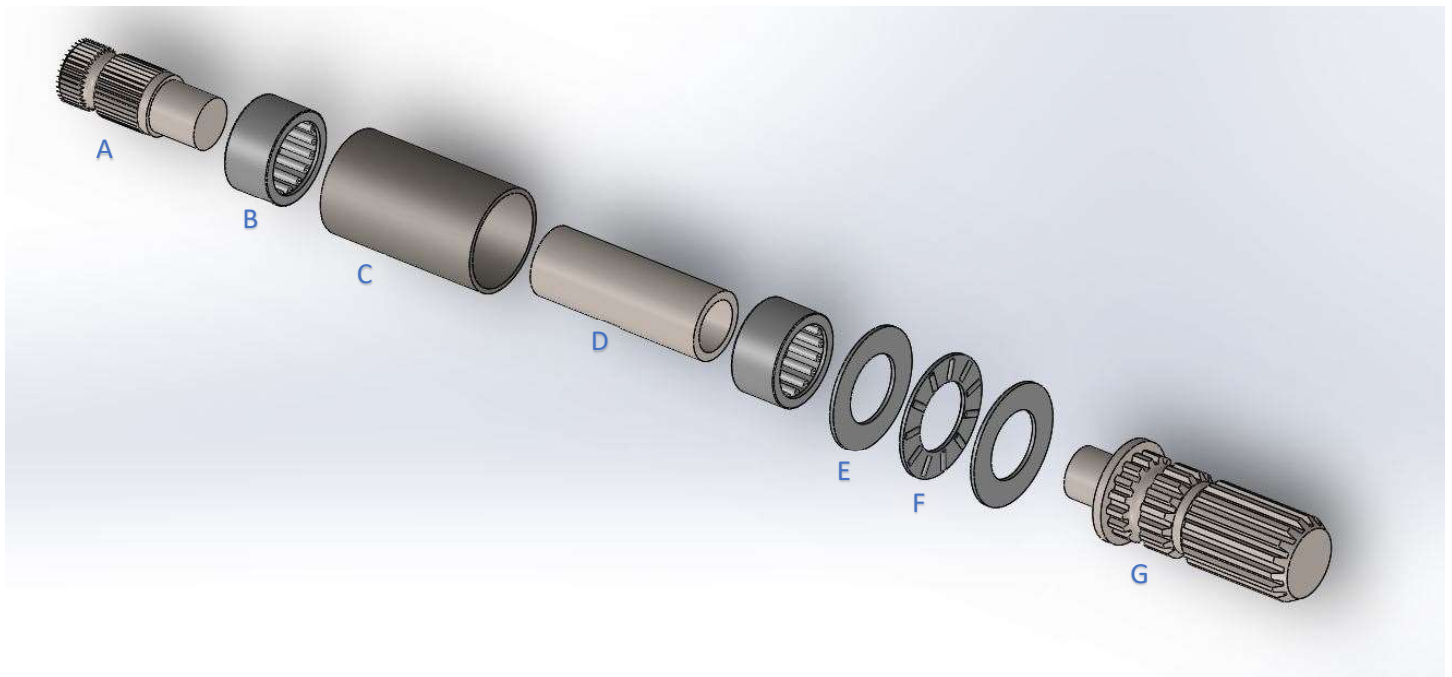


Figure 10: Upper steering column exploded view

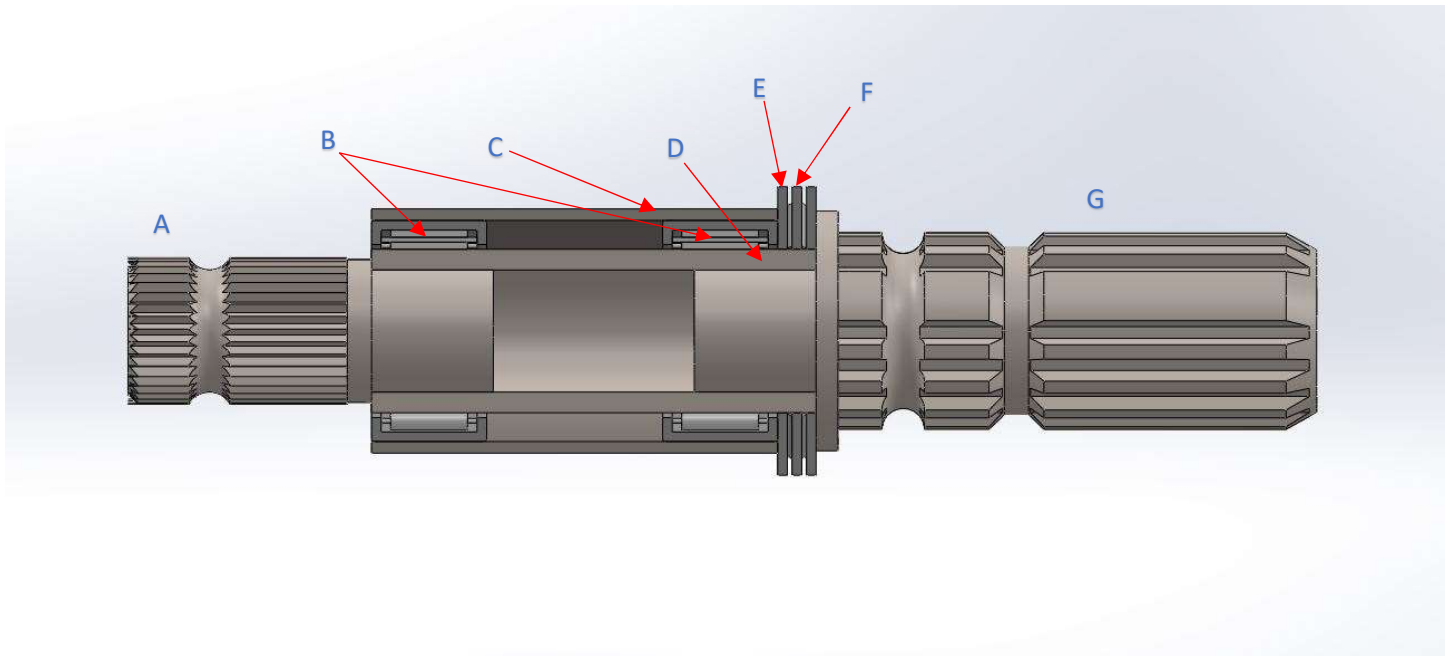


Figure 11: Upper steering column cross section view

Table 1: Upper Steering Column Bill of Materials

Part Label	Quantity	Part Description
A	1	Purchased .625"x36 splined stub, post processed in house to custom fit the upper steering column connector shaft
B	2	Needle roller bearing for 17mm shaft and 23mm housing diameter, McMaster part no. 5905k358
C	1	Frame support tube, 1" OD 4130 steel tube with a wall thickness of 0.035"
D	1	Upper steering column connector shaft, 17mm OD, 0.5" ID to match splined stubs
E	2	Thrust bearing washer for 17mm shaft, McMaster part no. 5909K74
F	1	Needle thrust bearing for 17mm shaft, McMaster part no. 5909K14
G	1	Purchased .75" quick release stub, post processed in house to custom fit the upper steering column connector shaft

The splined stub-connecting shaft joint was welded to ensure maximum stiffness, as well as to follow the FSAE guidelines for mechanical joints.

Lower Steering Column

Acting as the bridge between each u-joint, the lower steering column is the simplest component in the entire steering system. It simply consists of a tube with a splined shaft welded to each end to separate the two u-joints. In this design, the splined shaft is a 0.75" x 24 tooth spline, which is purchased, machined to size, then welded onto a tube.

The central tube can be analyzed with basic torsion stress and shear equations:

$$\frac{TL}{GJ} = \varphi \quad \frac{Tr}{J} = \tau_{max}$$

Where T = torque (in-lbs), L = length (in), G = shear modulus (psi), J = polar moment of inertia (in⁴), φ = angle of twist (°), r = radius (in), and τ_{max} = maximum shear stress. The maximum torque seen by the steering column can be calculated from the aligning torque of the tire, which can be found from the tire data in Figure 12 below [3].

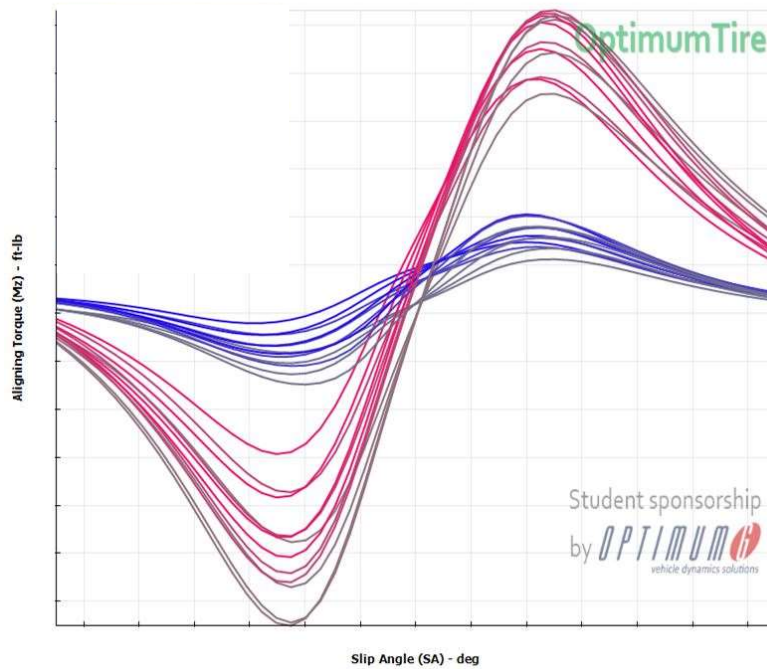


Figure 12: Aligning torque vs. slip angle of Hoosier 18.0x7.0-10R25b tires

The aligning torque was then changed to a force by dividing it by the length of the steering arm, then transformed back into a torque by multiplying the force by the steering pinion diameter. The final column geometry was then designed to simplify manufacturing by using stock tube sizes. The optimal size was determined by the lightest tube that gave a maximum twist angle of 0.01°, resulting in a stiff, yet light, lower steering column [4].

Steering Rack and Tie Rods

Steering Arm and Ackermann Introduction

The steering arm is possibly the most important geometrical choice in a conventional steering system, including FSAE and commercial vehicles. It is simply the moment arm that turns the wheel, but its placement has a drastic effect on the required force from the driver and the turning behavior of each wheel. Because a torque is equal to a force multiplied by a distance, for a given torque, the steering rack must place the necessary force on the steering arm to turn the wheel, depending on the steering arm's length. The other effect of the steering arm can be better understood with a prerequisite knowledge of Ackermann.

Ackermann

A vehicle with four wheels requires a special steering setup to drive efficiently. If a vehicle were to drive with what is known as parallel steering, at least one tire will scrub along the ground. This is because during a turn, the four tires are not turning about a common point. Instead, three tires are rolling along the ground, while one is

slipping. To prevent this, the Ackermann steering setup was invented. Ackermann steering adjusts the steering of each front wheel such that the inside wheel (inside of the turn) is turned more than the outside wheel. For a vehicle with a given wheelbase (l) and front track (w), the outside wheel turn (δ_o) for a given inside wheel turn (δ_i), or vice versa, can be found using the following equation:

$$\cot \delta_o - \cot \delta_i = \frac{w}{l} x$$

This equation is known as the Ackermann condition. If a given set of the four parameters satisfies the equation with $x=1$, that particular set qualifies as 100% Ackermann. However, no simple mechanical linkage configurations can follow the Ackermann condition perfectly throughout the steering motion, so Ackermann is usually stated as the value x that satisfies the equation for a particular set of wheel turn angles. In the FSAE Design Specifications Sheet, it is requested as the Ackermann percentage of the wheels turned to full lock at either side.

Figure 13 shows a steering setup with parallel steering, while Figure 14 shows a steering setup with 100% Ackermann.

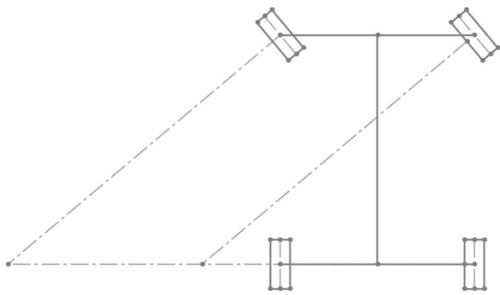


Figure 13: Parallel Steering Setup

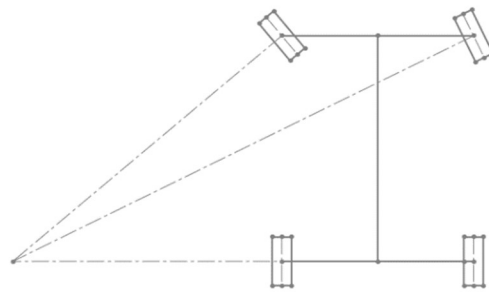


Figure 14: 100% Ackermann Steering Setup

With this information, it seems that 100% Ackermann is the most desirable steering setup since it results no tire slip at all four wheels. However, the effects of Ackermann steering become negligible as the speed of the vehicle increases due to the slip angles produced by the lateral force on the tires. The slip angle of a tire is an effect from both the lateral force placed on a rubber tire during a turn and the normal force on the tire. Simply put, it is the angle between the direction the tire is aligned and the direction the tire is traveling. Slip angle occurs because rubber is a flexible material, thus as the bottom of the tire, the contact patch, is rolling along the ground at a velocity of 0, the rest of the tire causes the bottom material to lag behind for any given lateral force on the tire. This slip angle just slightly reduces the lateral force placed upon the wheel, and therefore the body of the car. The lateral force that is not caught in the spring action of the rubber tire then goes into turning the body. Figure 15 shows a top view of the tire, with v representing the direction of tire travel and α representing the slip angle of the tire.

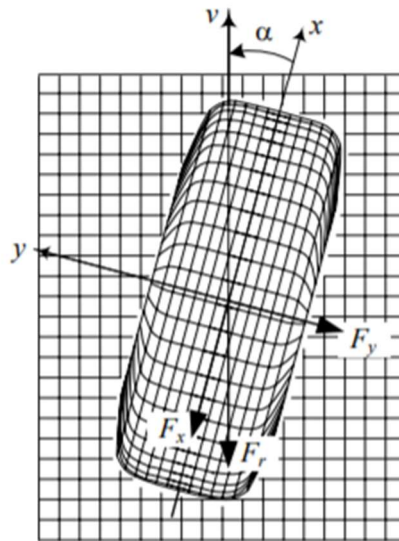


Figure 15: Slip angle of a rubber tire

Since these slip angles change with normal force on the tires, weight transfer across the car during the corner also dampens the effects of Ackermann steering. For these reasons, most passenger cars are set up for nearly parallel steering, while Formula 1 cars are set up with anti-Ackermann, or negative Ackermann. An anti-Ackermann setup causes the outside wheel to turn more than the inside wheel, and is necessary for vehicle control at the high speeds that Formula 1 sees. Figure 16 shows the effect that slip angles have on an Ackermann setup, with the solid lines representing the direction the tires, while the dashed lines leading from the wheels represent the direction the tires are actually traveling.

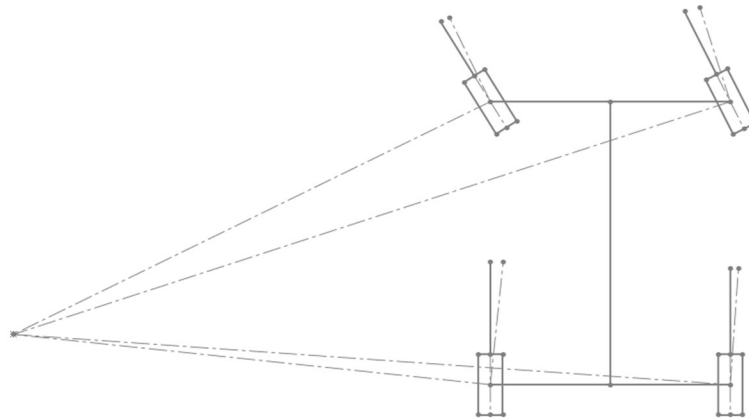


Figure 16: Ackermann steering setup with slip angles considered

It is important to know that Ackermann percentage is not necessarily a design constraint, it is a number calculated from the final design that is simply used to measure steered wheel turn ratios. As stated above, this number is given in the Design Specification Sheet as a way for the design judges to visualize the final steering system before seeing the final product in person.

For FSAE competitions, because of the low speeds and a non-zero weight transfer, an Ackermann percentage of 60-80% is best. This can be further narrowed based off of weight transfers, if known. However, this design will be based on clearances between the maximum wheel turn and the A-arms, which are static with reference to the turning wheel. The A-arms have been designed with maximum wheel turns of 33° for an inside wheel turn and 25° for an outside wheel turn. With the wheels going to these limits, we can calculate the Ackermann

percentage to be 75.4%, with our set wheelbase of 60.5" and front track of 48.5". Since this car will have minimal weight transfer due to anti-roll bars, an Ackermann percentage of 70-80% is desirable.

Steering Rack Choice

Steering Rack Placement

The placement of the steering rack has a great effect on the turning behavior of the tires. This placement works with the steering arm geometry to define the specific Ackermann setup. From an in-wheel packaging perspective with pro-Ackermann, it is the most beneficial to place the steering rack behind the front wheel centerline. This allows the outboard tie-rod location to be away from any features inside the wheel by moving the tie-rod point inboard. This benefits packaging by giving the tie rod clearance from the turning wheel.

Support Placement along Rack

The forces seen on a steering system are non-negligible and can induce compliance within the system, which leads to poor road feedback for the driver and unpredictable loading in the components. One of the easier ways to counteract these forces is simply to mount the steering rack at the correct locations. The steering rack sees forces from two places: the driver and the wheels. Therefore, the mounting system for the rack must provide the necessary reaction forces to counteract the driver and wheel forces.

By turning the wheel, the driver imparts a torque upon the rack as a whole through the pinion. If it is assumed that the rack and pinion are locked in place and rotation, this torque will try to turn the body of the steering rack. Therefore, the mounting system needs to provide a counter-torque. In the 2019 WUFR design, the mounting points for the rack are placed as far from the pinion as possible, thus minimizing the force placed upon each mounting point. The benefit of having two mounting points is that they provide a couple moment, which only imparts a rotational force on the body of the rack without translation. If the mounting points of the rack were placed close to the center of rotation, there would be a significant displacement at the ends of the rack.

When the rack experiences force from the tires via the tie rods, the forces are approximately axial along the rack. Most of this axial force is transferred through the rack body and into the opposite wheel, and the small non-axial forces are placed upon the end of the rack. Due to placing the mounts close to the ends of the steering rack, the mounts are in the correct place to directly counteract any of these small forces. If the tie rods were placed with greater misalignment with the rack, the force components in the radial direction of the rack would need to be accounted for.

Total Pinion Travel and Rack Ratio

These parameters are set by the available steering racks, and define how much force and rotation the driver must place on the steering wheel to rotate the front wheels. With a given steering arm and given rotation of the steering wheel, the amount that the steered wheel turns is determined by the rack travel from the given steering wheel rotation. This amount of rack travel is based on the pinion diameter in the rack and pinion system: a larger pinion would mean more rack travel for a given rotation. However, purchased steering racks only come in limited pinion diameters.

Available Racks

Table 2 below shows the critical specifications for the available steering racks.

Table 2: Steering Rack Specifications

Steering Rack	Brand	Weight (lb)	Total Length (in)	Travel per Side (in)	Total Rotation (deg)	Pinion Diameter (in)	Travel per Degree of Rotation
C42-336	Stiletto	1.96	8.5	1.5	405	0.85	0.00741
C42-340	Stiletto	2.34	11.25	2.3125	630	0.84	0.00734
Zrack358	Zrack	0.77	14.095	0.988	210.8	1.07	0.00937
KazRack	Kaz	3	15.225	1.625	246	1.51	0.01321

Table 3 below shows the best available steering racks, with their features and specifications scored to give an approximate overall ranking.

Table 3: Steering Rack Selection Criteria

									
		Stiletto C42-336		Stiletto C42-340		Zrack358		KazRack	
Selection Criteria	Weighted %	Rating	Weighted	Rating	Weighted	Rating	Weighted	Rating	Weighted
Mounting System	40	1	0.4	1	0.4	5	2	5	2
Max Pinion Rotation	20	2	0.4	3	0.6	4	0.8	4	0.8
Rack Travel per Side	15	3	0.45	4	0.6	1	0.15	3	0.45
Total Length	10	1	0.1	2	0.2	4	0.4	4	0.4
Weight	10	3	0.3	3	0.3	5	0.5	2	0.2
Price	5	5	0.25	5	0.25	1	0.05	3	0.15
Total Score		1.9		2.35		3.9		4	
Rank		4		3		2		1	

The KazRack boasts a solid mounting system and comparable specifications to the Zrack358, and a reasonable amount of rack travel. The amount of rack travel translates to a lower driver rotational force to rotate the tires due to a longer available steer arm length. While it is heavy compared to the other choices, the KazRack is durable at an affordable price. For these reasons, the KazRack will be the central component of this steering assembly.

Steering Axis

The steering axis is the axis about which a steered wheel turns. It is the fulcrum that the steering arm rotates about, and is defined with relations to the ground plane and tire contact patch, which is the point where all the resultant forces on a tire occur.

Geometries

The parameters that define the steering axis have a huge effect on driver feel, steering force, tire wear, and bump steer. The most important geometries are kingpin inclination, scrub radius, caster, mechanical trail, and camber. These variables are defined through one of two reference planes: the front view plane and the side view plane. These view planes and geometries are shown below in Figure 17 [1].

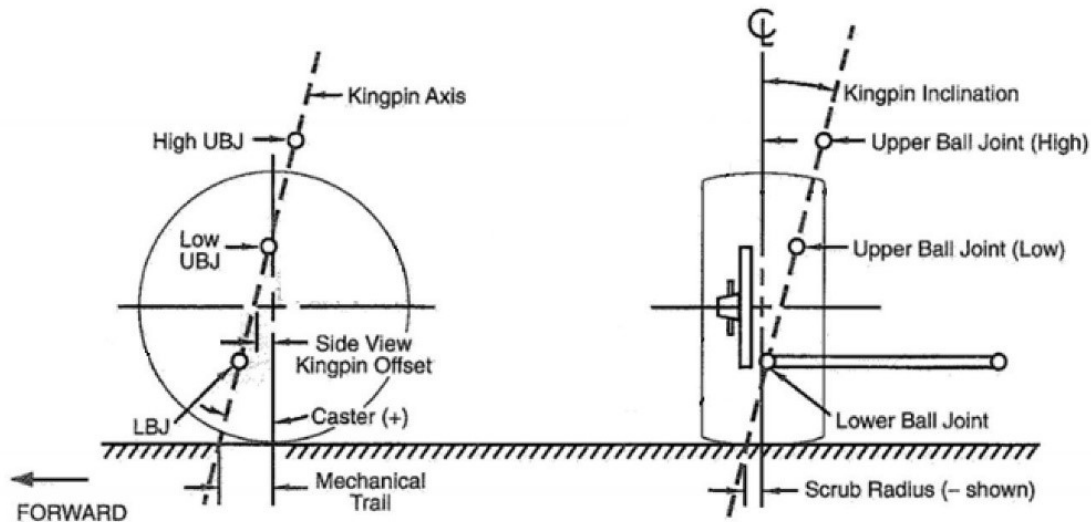


Figure 17: Steering axis geometries

Caster and Kingpin Inclination Angle

These two angles, caster and kingpin inclination (KPI), are the angles at which the steering axis is tilted in the side and front view planes, respectively.

Caster and KPI each affect the body of the car while steering: caster tends to lift the body of the car when the front of the wheel is steered away from the body, and drop the body as the front of the wheel is steered towards the car. KPI, however, lifts the body of the car no matter which direction the wheels are steered. Since there is the constant force of gravity upon the body, a high KPI will cause a small self-centering effect at low speeds.

Mechanical Trail and Scrub Radius

These lengths place the steering axis in the correct location relative to the center of the tire contact patch, which is the resultant point at which all the forces upon the tire act. The mechanical trail is the distance between the center of the tire contact patch and the point the steering axis meets the ground, all in the side view. It is defined as positive when this point is in front of the center of the tire's contact patch. This small system can be thought of as a pendulum with positive trail acting as a conventional pendulum, providing wheel stability; negative trail is this same pendulum but flipped over, therefore causing disastrous effects on wheel stability. Mechanical trail supplements caster's self-centering effect; a higher trail will increase the caster effects so that a higher force is needed to turn the wheels. A lower trail allows the tires to be more easily steered, and a wheel with zero mechanical trail can be steered with almost no resistance from lateral force on the wheels. However, this results in a complete lack of driver feel, so a small amount of mechanical trail is necessary for proper driving.

The scrub radius is the distance from the center of the contact patch to the point at which the steering axis intersects the ground in the front view. It is defined as negative when the distance is outside the car, away from the body. Scrub acts as the moment arm for any longitudinal forces wishing to turn the wheels, leading to a higher amount of bump steer (undesirable steer that occurs as the wheels travel vertically) for a higher scrub radius. However, a car with zero scrub radius feels "dead" to drive, since there is no information transferred into the steering wheel from the road.

Camber

Camber is the angle at which the wheels are inclined in relation to the ground from the front view plane. It is measured from the vertical axis and is defined as positive when the top of the wheels are tilted away from the

car. Due to the KPI and caster angles, camber changes for each of the four types of chassis movement: pitch, heave, roll, and steer. Pitch is the rotation of the chassis about the lateral axis at the center of gravity. This results in the chassis front dipping down under braking (known as dive), or the front rising under acceleration (known as squat). Roll is the rotation of the chassis about the longitudinal axis at the center of gravity, and only happens when turning. Roll and pitch are each directly proportional to the height of the center of gravity from the ground. Heave is the vertical translation of the chassis up or down due to body forces. This effect happens mainly in cars designed with aero-packages, where the downforce produced from the wings and undertray acts as the body force. Steer, of course, is the rotation of the wheels used for changing the vehicle direction. Camber change in steer is influenced by the placement of the steering axis relative to the wheel, which is described by the kingpin inclination and the caster. The amount of steer camber under these variables can be described by the formula below:

$$\text{Steer Camber} = KPI * (1 - \cos\theta) - \text{Caster} * \sin\theta + SC$$

Where θ is wheel turn in degrees, and SC is static camber in degrees. Camber direction is extremely important for tire grip, especially in the non-radial tires used for racing. Non-radials are used because the structure of the rubber leads to extra lateral force resistance when negatively cambered. Racing tires can be statically cambered up to -4° to both counteract the camber gain due to caster and produce more grip when encountered with a lateral force. However, as a warning, non-radial tires should never positively camber as they lack supporting side structures within the tire.

Steering Arm

The most critical geometric feature in the outboard steering system is the steering arm. The steering arm is defined as the moment arm through which the tie rod acts upon the steering axis. The steering arm is not a physical part of the wheel assembly: it is a load path that makes the wheels turn. The vertical location of the steering arm-tie rod point determines the type of loading seen by the tie rod, while the XY location of the point sets the Ackermann behavior at any point within the wheel turn.

Smallest Possible Turn Radius

The goal of the steering arm is to allow for the car to steer in the tightest circle that it will see on the track. For our purposes, this steering design will be calibrated such the car can execute a turn with an outside diameter of 9 meters, which is the smallest turn that will be seen in competition according to the rules. This turn is a hairpin style turn, meaning the car will travel at least a full 180° about, in this case, a single point.

We still need to account for a maximum trackwidth in the front of 48.5 inches, measured from the centers of the contact patches of the front right and front left tire; the tire width of 7.5 inches; and the minimum track width of 3.5 meters. From the track and tire width, we can calculate that our vehicle's centerline is placed 28 inches from the outside wall of the outside wheel, or the outside wall of the inside wheel. The shortest path around the turn would, of course, be hugging the inside wall. However, the shortest path is not always the fastest path when concerning a racecar.

The slowest parts of any race are the corners. This is because the drivers must slow down to maintain grip throughout the turn, then accelerate out of the turn to return to racing speeds. The amount that the driver needs to slow down is also directly related to the radius of the path around the corner: if the turn radius is small, the speed at which the turn is taken needs to be lower.

When vehicle acceleration and braking are accounted for in this turn, the quickest path would be a half circle tangent to the outer walls before and after the turn, and tangent to the inside wall at the apex of the turn. The apex of the turn is chosen by the driver and depends on the previous and future directions of the track. But for a turn similar to the infamous hairpin of FSAE Michigan 2018, the apex would be along the inside wall in the middle of the turn. These conditions are were used to calculate BFR19's turn radius.

The result is that the quickest path for the car correlates with a turn radius (measured to the car's center) of 3.79 meters, or 149.2 inches. However, to account for driver error, the calculations will be run with a smaller turning radius as a conservative measure. It is impossible to determine this exact tolerance without the drivers becoming familiar with the WUFR19, so this tolerance is based on the drivers' own self-assessed ability to follow the correct line. Based upon driver interviews, it will be assumed that the drivers can follow the perfect line within 6 inches. Including this 6 inch clearance, the turning radius which will be designed for is 3.64 meters, or 143.2 inches.

Physically achieving this turning radius is determined by many factors, including vehicle trackwidth and wheelbase, tire dynamics, and steering geometry. The trackwidth and wheelbase are set according to other suspension designs, and calculations can be done to predict tire dynamics. The final parameter, steering geometry, is the most variable thing available, so the geometry of the steering arm will be used to achieve this turning radius.

Wheel Turn Angles

The wheel turn angles which will be used in the design will be 33° for a maximum inside wheel turn and 25° for a maximum outside wheel turn. This was determined from packaging constraints with the A-arms and the steered wheels, as the A-arms will contact the wheel rims as the wheels are steered [8].

Turn Radius Calculations

The turning radius of a vehicle can be calculated using the following equation [5]:

$$R = \sqrt{a^2 + l^2 \left(\frac{\cot \delta_o + \cot \delta_i}{2} \right)^2}$$

Where R is turn radius, a is the distance from the rear axle to the center of mass of the car, l is the wheelbase of the car, δ_o is the outer wheel turn angle, and δ_i is the inner wheel turn angle. Using the previously discussed wheel turn angles, $l = 60.5$ in, and $a = 30$ in, the final turn radius is 115.5 inches. However, this radius is measured from the center of the mass of the car, which means half the track must be added to find what the minimum turn can be. Adding 24.25 inches leads to a minimal turn of 140 inches, or 3.6 meters. However, this does not account for inner and outer tire slip angles, which can be assumed to be 3° and 5° respectively. When these are accounted for, the final minimum outside radius is 163 inches, or 4.2 meters, which is perfect when accounting for driver error.

Vertical Tie Rod Location

The vertical location of the outboard tie rod point influences the load type placed on the tie rod. This vertical location is wherever the XY point lies along one of two lines. The first lines, the blue lines in Figure 18, which is a front view of the front wheel centerline plane, are the conventional tie rod point location. This location is in plane with the steering rack such that no vertical movement. However, if the tie rod is placed along this line, if both wheels were to move vertically, therefore rotating about their individual instantaneous centers, the tie rod would resist this vertical motion at the outboard point. This would result in possible bump steer, or even tie rods placed in bending. However, if an approach similar to the other suspension linkages is taken by aligning the tie rods along the line from the steering rack (red line) point to the respective wheel instantaneous center (the green lines), this bending force can be removed entirely in straight line driving with rising and falling tires.

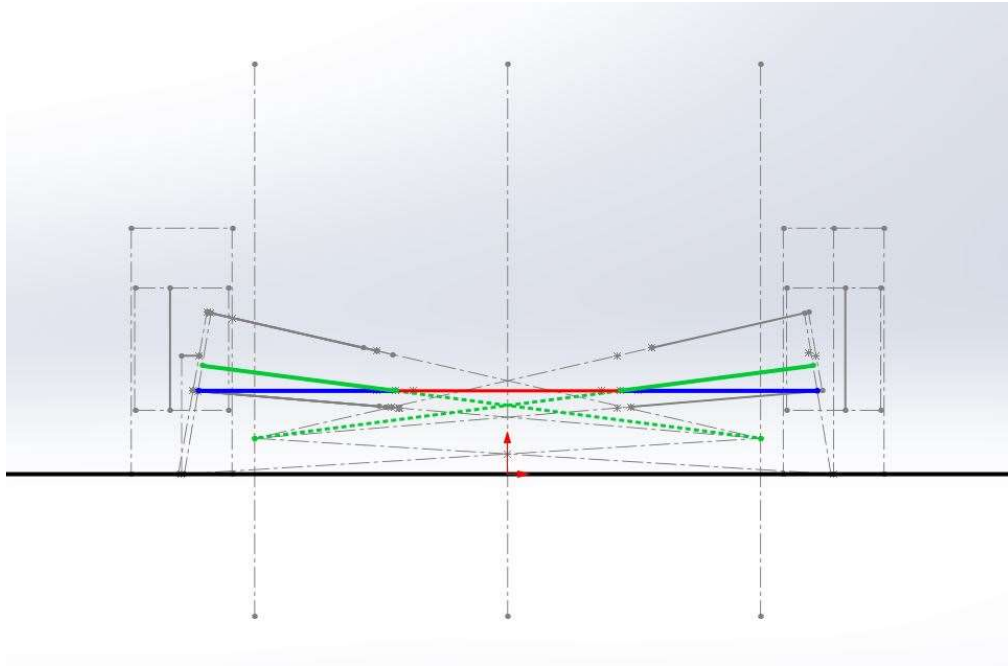


Figure 18: Possible tie rod front view locations

Ackermann Revisited

The desired wheel turn angles define the placement of the ourboard tie rod point from the top view. For an easier understanding, the lower steering system (steering rack, tie rods, steering arms, etc.) can be modeled as a trapezoidal four bar linkage, as shown in Figure 19 in an unsteered configuration, and Figure 20 in a steered configuration [2].

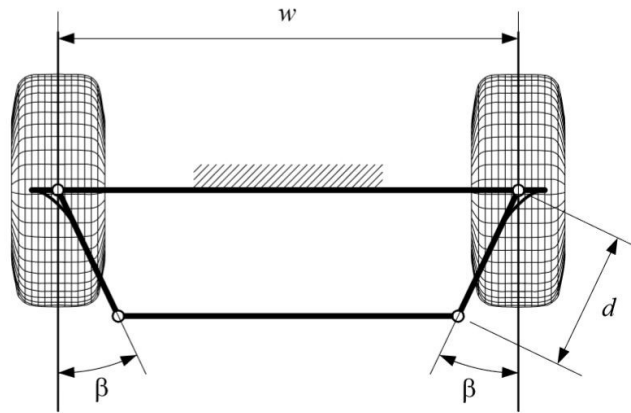


Figure 19: Trapezoidal steering mechanism

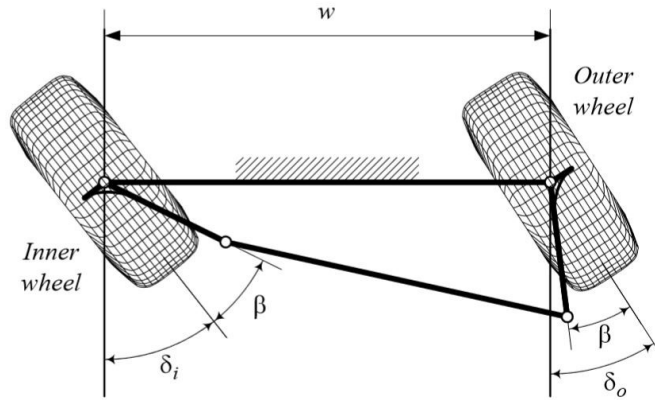


Figure 20: Trapezoidal steering mechanism (steered)

One of the most important variables shown in Figures 19 and 20 is β , which is the angle between the wheel side plane and the steering arm. This angle allows the inner wheel to turn more or less than the outside wheel while being steered, depending on the tie rod position within the wheel. Figure 21 below shows the effect on the Ackermann for various points in relation to the wheel.

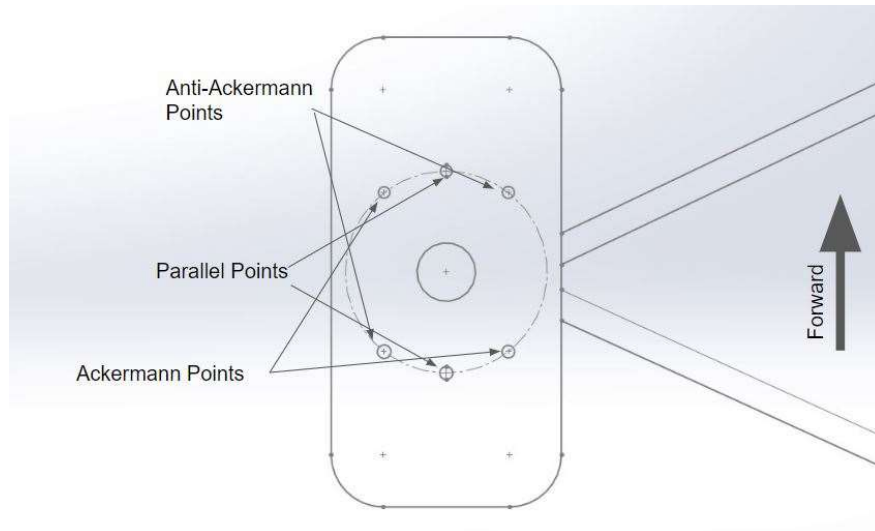


Figure 21: Possible in-wheel tie rod locations

As mentioned before, one of the largest drawbacks to using mechanical systems to turn wheels is that a mechanical system, especially a four-bar linkage, cannot model an Ackermann system at every position. This variance from 100% and the design Ackermann, 79%, can be modeled with relation to the angle β , as seen in Figure 22 below (see Appendix C for source code).

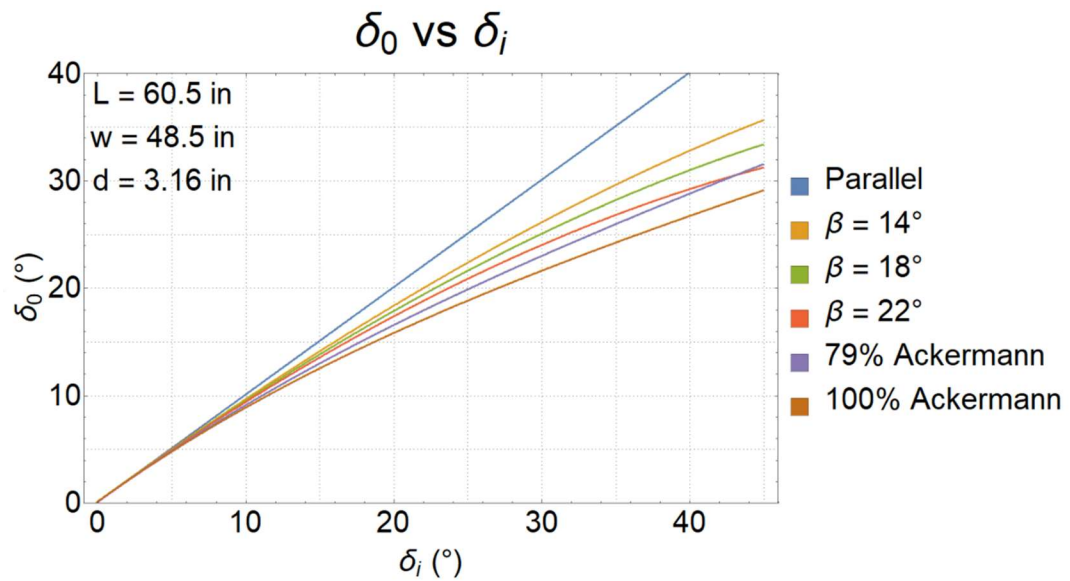


Figure 22: Comparison between theoretical and mechanical steering setup

For this design, the rearward ackerman point was chosen due to a better packaging opportunity. To determine its exact location, three similar static assembly models of a single side of the steering system was used. These assemblies included wheels and uprights. In these static assemblies, an initial steering arm was placed, which had estimated dimensions which would be edited to achieve the final goal. These static models were connected into one by the steering axis in each model, resulting in only one rotational degree of freedom. This degree of freedom was eliminated by inserting three tie rods to connect at three points along the steering rack and the steering arm in each of the three assemblies. This gave a final static assembly which was able to provide measurements for inner and outer wheel turn angles. The next step was varying the lengths of the tie rods and the steering arm geometry until the desired wheel turn angles of 33° (outer) and 25° (inner) were found. The final assembly is shown below in Figure 23 [7].

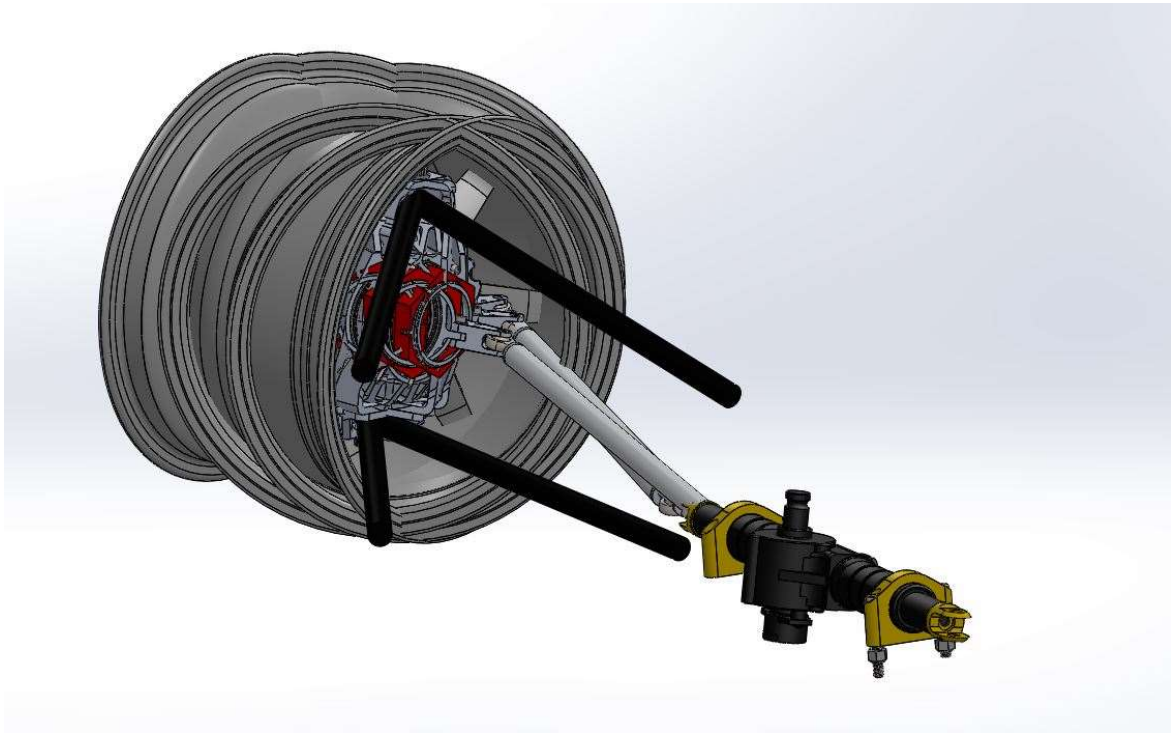


Figure 23: Static model for steer angle measurements

But why use a static CAD model rather than a dynamic one? The largest reason is that a static model is both simpler and requires less computing time, while a dynamic model must have the correct permanent constraints and is generally more unreliable due to the large amount of processing that is required. Another huge benefit of the static model is that any component that might inhibit wheel turn, such as A-arm bearing housings, could easily be imported into the assembly then checked for clearances. However, if there was sufficient knowledge in motion studies and enough computing power, a dynamic model would be much more adjustable.

Final Design Execution

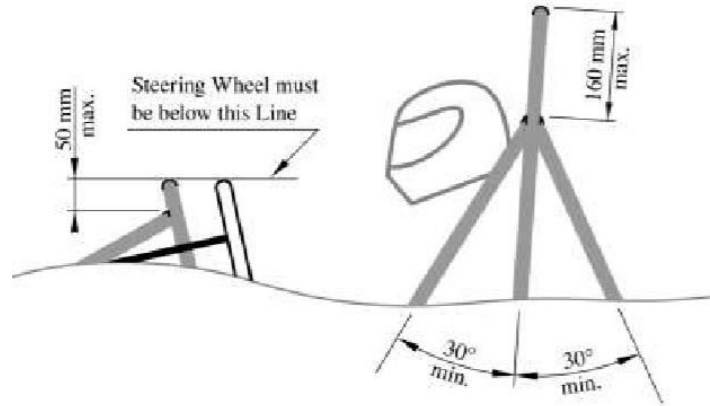
The final design of this system is not necessarily the most optimal steering system that could be made with the given design goals and limitations, but the research and experience gained from this project will be a helpful information source until drastic design changes are made. Going forward, this design should be done alongside the design of the uprights due to packaging constraints, which could be reduced if this action was taken.

Appendices

Appendix A: Supplementary Texts

Supplementary Rules

T.2.13.4 The Main Hoop braces must be attached as near as possible to the top of the Main Hoop but not more than 160 mm below the top-most surface of the Main Hoop. The included angle formed by the Main Hoop and the Main Hoop braces must be at least 30°.



T.10.2 Critical Fastener Requirements

T.10.2.1 Any Critical Fastener must meet, at minimum, one of the following: a. SAE Grade 5 b. Metric Grade 8.8 c. AN/MS Specifications

T.10.2.2 All Critical Fasteners must be one of the following:

- 1) Hex head, or
- 2) Hexagonal recessed drive (Socket Head Cap Screws or Allen screws/bolts)

T.10.2.3 All Critical Fasteners must be secured from unintentional loosening by the use of Positive Locking Mechanisms.

T.10.2.4 Some Critical Fastener applications have additional requirements that are provided in the applicable section

Positive Locking Mechanisms

T.10.3.1 Positive Locking Mechanisms are defined as those which:

- a. The Technical Inspectors (and the team members) are able to see that the device/system is in place (visible).
- b. The Positive Locking Mechanism does not rely on the clamping force to apply the locking or anti vibration feature. (If it loosens a bit, it still prevents the nut or bolt coming completely loose)

T.10.3.2 Acceptable Positive Locking Mechanisms include:

- a. Correctly installed safety wiring
- b. Cotter pins
- c. Nylon lock nuts (where temperature does not exceed 80°C)
- d. Prevailing torque lock nuts

Lock washers, bolts with nylon patches and thread locking compounds (Loctite®), DO NOT meet the positive locking requirement.

Appendix B: MATLAB Code for Universal Joint Graphs

```
%% Variables
angular_position = linspace(0,360);
gamma = [0,5,10,15,20,25,30];
input_w = 1;
c = 90;
misalignment = [0,30,45,60,75,90];

%% Creating Arrays
% Arrays for various axis misalignments
for j = 1:7
    for i = 1:100
        output1(i,j) = ujoint(input_w, gamma(j), angular_position(i));
    end
end

% Single array for comparison
for i = 1:100
    output_20deg(i) = output1(i,5); % creating single array
end

% Arrays for various input misalignments
for j = 1:6
    for i = 1:100
        output2(i,j) =
ujoint(output_20deg(i), gamma(5), angular_position(i)+misalignment(j));
    end
end

%% Plots
figure;
plot(angular_position, output1)
axis([0 360 0.85 1.2])
lgd = legend('0°', '5°', '10°', '15°', '20°', '25°', '30°')
title(lgd, 'Axis Misalignment')
ylabel('Ratio of Angular Velocities')
xlabel('Input Shaft Angular Position (°)')
title({'Angular Velocity Ratio of Single U-Joint'; 'with Varying Axis Misalignment'})
```

```

figure;
hold on;
axis([0 360 0.85 1.2])
plot(angular_position,output_20deg,'--')
plot(angular_position,output2)
lgd2 = legend('Original Input','0°','30°','45°','60°','75°','90°')
title(lgd2,'Input Misalignment')
ylabel('Ratio of Angular Velocities')
xlabel('Initial Input Shaft Angular Position (°)')
title({'Angular Velocity Ratio of Two U-Joints in Series';'with Varying  
Input Misalignment';  
'(Axis Misalignment of 20°)'})
hold off;

%% Functions
function output_w = ujoint(input_w,gamma,theta)
    output_w = (cosd(gamma)/(1-(sind(gamma)^2*cosd(theta)^2)))*input_w;
end

```

Appendix C: Mathematica Code for Inner and Outer Wheelturn Angles

(*Look on page 387 of Steering Dynamics Third Edition by Reza Jazar for explanation*)(*Steering by Connor McRae. Code by Jeff Ahlers*)

L=1.5367;(*m*) (*They're in meters but it shouldnt matter. They're all ratios.*)

w=1.2319;(*m*) (*If you change these parameters and want to run g[] in the second section, you have to run the first section first*)

d=0.080264;(*m*)

(*β is in Degrees*)

dnf=.5; (*Granularity of the graph. In degrees. Gives a nice graph around .05 ish*)

LHS[δi_,δo_,β_]:=Sin[β+δi]+Sin[β-δo];

RHS[δi_,δo_,β_]:=w/d-Sqrt[(w/d-2Sin[β])^2-(Cos[β-δo]-Cos[β+δi])^2];

Ack[δi_,w_,L_,per_]:=180/π*ArcCot[Cot[δi]+(per*w)/L];

Parallel[δi_]:=Piecewise[{{δi,δi<40}},Null];

(*This solves for δo from a given δi. Most of this is just making the data useable. Try doing the Solve[] section with constants (except δo) to see*)

(*Select bounds are in radians*)

f[δi_,β_]:=180/π*(First[Select[Select[Re[δo/.Solve[LHS[δi,δo,β*π/180]==RHS[δi,δo,β*π/180],δo]],#>-.001&],#<1.4&]]];

(*δi,β, and δo are inputted/outputted as degrees but converted to degrees for calculations*)

DiscretePlot[{Parallel[δi],f[δi*π/180,14],f[δi*π/180,18],f[δi*π/180,22],Ack[δi*π/180,w,L,.79],Ack[δi*π/180,w,L,1]}, {δi,0,45,dnf},PlotLegends->SwatchLegend[{"Parallel", "β = 14°", "β = 18°", "β = 22°", "79% Ackermann", "100% Ackermann"},LegendMarkerSize->25],(*Add new curves before here*)

```

PlotLabel->Style["Subscript[ $\delta$ , 0] vs Subscript[ $\delta$ , i]",FontSize->60,Black],Filling->None,FrameLabel->{"Subscript[ $\delta$ , i] (°)", "Subscript[ $\delta$ , 0] (°)"},LabelStyle->Directive[Black, FontSize->40],Frame->True,GridLines->{5*Range[#2]&, 5*Range[#2]&},GridLinesStyle->Directive[Dotted, Gray],ImageSize->Large,Epilog->Text[Style["L = 60.5 in \nw = 48.5 in \n d = 3.16 in ", FontSize->40],{5,34}], PlotMarkers->Style["•", FontSize -> 9],PlotRange->{0,40}]
dng=.5;
g[ $\delta_i$ _, $\beta$ _,w_,L_,per_]:=f[ $\delta_i$ , $\beta$ ]-Ack[ $\delta_i$ ,w,L,per]
DiscretePlot[{g[ $\delta_i$ * $\pi$ /180,18,w,L,.79],g[ $\delta_i$ * $\pi$ /180,18,w,L,1]},{ $\delta_i$ ,0,45,dng},PlotLegends->SwatchLegend[{"79% Ackermann,  $\beta = 18^\circ$ ", "100% Ackermann,  $\beta = 18^\circ$ "},LegendMarkerSize->25],(*Add new curves before here*)

PlotLabel->Style["Subscript[ $\Delta\delta$ , 0] vs Subscript[ $\delta$ , i]",FontSize->60,Black],Filling->None,FrameLabel->{"Subscript[ $\delta$ , i] (°)", "Subscript[ $\Delta\delta$ , 0] (°)"},LabelStyle->Directive[Black, FontSize->40],Frame->True,GridLines->{5*Range[#2]&, 5*Range[#2]&},GridLinesStyle->Directive[Dotted, Gray],ImageSize->Large,Epilog->Text[Style["L = 60.5 in \nw = 48.5 in \n d = 3.16 in ", FontSize->40],{6,3.5}], PlotMarkers->Style["•", FontSize -> 9],PlotRange->Automatic]

```

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